Please write clearly in block capitals.

Centre number | | | | | |  Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____
I declare this is my own work.

# AS
# FURTHER MATHEMATICS

## Paper 1

Monday 12 May 2025 Afternoon Time allowed: 1 hour 30 minutes

**Materials**
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
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| 16 | |
| **TOTAL** | |

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Answer **all** questions in the spaces provided.

**1** Calculate the product

$$(3 + \mathrm{i})(2 - \mathrm{i})$$

Circle your answer.

**[1 mark]**

$5 - \mathrm{i}$ $5 + \mathrm{i}$ $7 - \mathrm{i}$ $7 + \mathrm{i}$

**2** The complex number $3 + 5\mathrm{i}$ is a root of the quadratic equation

$$z^2 + az + b = 0$$

where $a$ and $b$ are real constants.

Find the other root of the equation.

Circle your answer.

**[1 mark]**

$3 - 5\mathrm{i}$ $3 + 5\mathrm{i}$ $5 - 3\mathrm{i}$ $5 + 3\mathrm{i}$

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**3** Find the equations of the asymptotes of the curve with equation

$$y = \frac{(x-1)(x+2)}{(x+1)(x-2)}$$

Tick (✓) **one** box.

**[1 mark]**

$x = 1,\ x = -2,\ y = 1$ ☐

$x = 1,\ x = -2,\ y = -1$ ☐

$x = -1,\ x = 2,\ y = 1$ ☐

$x = -1,\ x = 2,\ y = -1$ ☐

**4** Find the first two non-zero terms of the Maclaurin series expansion of $\cos(2x)$

Tick (✓) **one** box.

**[1 mark]**

$1 - x^2$ ☐

$1 - 2x^2$ ☐

$2 - x^2$ ☐

$2 - 2x^2$ ☐

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**5** The quartic equation

$$3x^4 + x^2 - 5x - 11 = 0$$

has roots $\alpha, \beta, \gamma$ and $\delta$

**5 (a)** Write down the value of $\alpha\beta\gamma\delta$

**[1 mark]**

**5 (b)** Write down the value of $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

**[1 mark]**

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**6** The complex numbers $w$ and $z$ are defined as follows:

$w = 2(\cos 0.4 + \mathrm{i}\sin 0.4)$ and $z = 6(\cos 1.2 + \mathrm{i}\sin 1.2)$

**6 (a)** Express $z^2$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$

**[2 marks]**

**6 (b)** Express $\frac{z}{w}$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$

**[2 marks]**

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**7 (a)** Geraldine wants to show that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

She writes her steps as follows:

$$\begin{aligned}
&\text{Let} \quad y = \tanh^{-1} x \\
&\Rightarrow \quad \tanh y = x \\
&\Rightarrow \quad \frac{e^{y} + e^{-y}}{e^{y} - e^{-y}} = x \\
&\Rightarrow \quad e^{y} + e^{-y} = xe^{y} - xe^{-y} \\
&\Rightarrow \quad (1+x)e^{-y} = (x-1)e^{y} \\
&\Rightarrow \quad e^{2y} = \frac{1+x}{x-1} \\
&\Rightarrow \quad 2y = \ln\left(\frac{x+1}{x-1}\right) \\
&\therefore \quad \tanh^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)
\end{aligned}$$

Identify and explain the error in Geraldine's method.

**[2 marks]**

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**7 (b)** Use the correct identity to find

$$\tanh^{-1}\left(-\frac{24}{25}\right)$$

Give your answer in the form $\ln a$ where $a$ is a rational number.

**[2 marks]**

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**8 (a)** Show that, for all positive integers $r$,

$$\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{2r+1}{r^2(r+1)^2}$$

**[1 mark]**

**8 (b)** Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = \frac{an^2+bn}{(n+1)^2}$$

where $a$ and $b$ are integers to be found.

**[3 marks]**

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**8 (c)** Hence show that, for all positive integers $c$,

$$\sum_{r=c}^{2c} \frac{2r+1}{r^2(r+1)^2}$$

can be written in the form

$$\frac{(pc+1)(c+1)}{c^2(qc+1)^2}$$

where $p$ and $q$ are integers to be found.

**[3 marks]**

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Do not write outside the box

**9** Solve the inequality

$$\frac{2x+1}{3x-9} \geq x-1$$

**[4 marks]**

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**10** The $3 \times 3$ matrix **M** represents a reflection in the plane $z = 0$

The point $A$ has position vector $\begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$

The point $A'$ is the image of $A$ under the reflection represented by the matrix **M**

Line $l$ passes through $A$ and $A'$

**10 (a)** Write down the matrix **M**

**[1 mark]**

**10 (b)** Find the position vector of $A'$

**[1 mark]**

**10 (c)** Write down a vector equation of $l$

**[2 marks]**

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**10 (d)** The point $B$ has coordinates $(3, 7, -2)$

**10 (d) (i)** Write down the coordinates of the point on $l$ which is closest to $B$

**[1 mark]**

**10 (d) (ii)** Calculate the shortest distance between $l$ and $B$

**[1 mark]**

**10 (d) (iii)** Calculate the area of the triangle $ABA'$

**[2 marks]**

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**11** The $2 \times 2$ matrix **A** represents an anticlockwise rotation of 150° about the origin.

**11 (a)** Find matrix **A**

Write each element in its simplest exact form.

**[2 marks]**

**11 (b)** Find matrix $\mathbf{A}^2$

Write each element in its simplest exact form.

**[2 marks]**

**11 (c)** Fully describe the transformation represented by the matrix $\mathbf{A}^5$

**[2 marks]**

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**11 (d)** Find the least positive integer $n$ which satisfies the equation

$$\mathbf{A}^n = \mathbf{I}$$

**[3 marks]**

**Turn over for the next question**

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**12** Use the definitions of $\sinh x$ and $\cosh x$ in terms of $e^x$ and $e^{-x}$ to solve

$$5\sinh x - 3\cosh x = 6$$

Give your answer in the form

$$x = \ln\left(a + \sqrt{b}\right)$$

where $a$ and $b$ are integers.

Fully justify your answer.

**[6 marks]**

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**Turn over for the next question**

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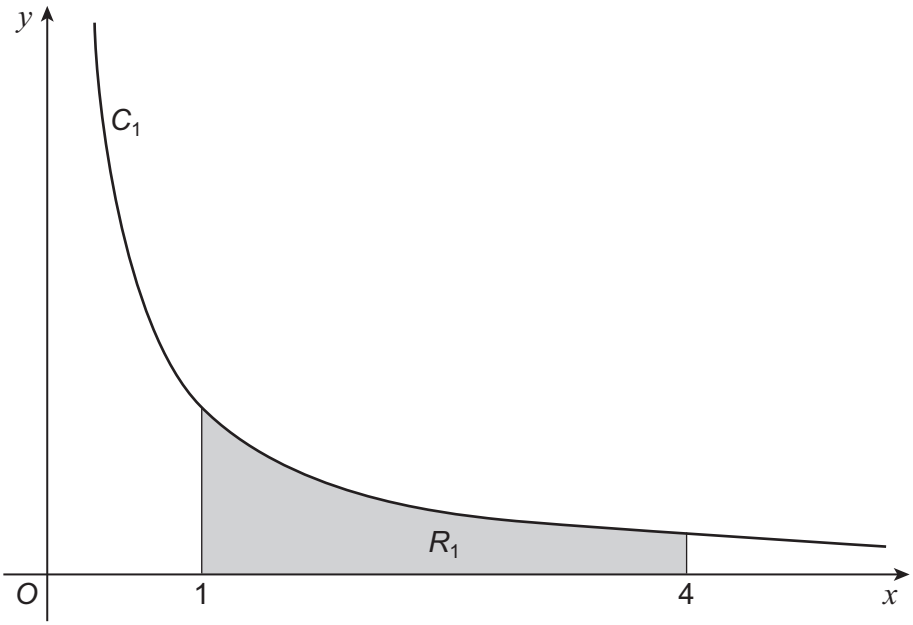
**13** The curve $C_1$ is given by the equation

$$xy = m$$

where $m$ is a positive constant.

The region $R_1$ is enclosed by curve $C_1$, the $x$-axis and the lines $x = 1$ and $x = 4$




The region $R_1$ is rotated through $2\pi$ radians about the $x$-axis.

The volume of the solid generated is $V$

**13 (a)** Find a simplified expression for $V$ in terms of $m$

**[2 marks]**

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**13 (b)** The curve $C_2$ is a stretch of $C_1$ by a factor 5 in the $y$-direction.

The region $R_2$ is enclosed by $C_2$, the $x$-axis and the lines $x = 1$ and $x = 4$

The region $R_2$ is rotated through $2\pi$ radians about the $x$-axis.

The volume of the solid generated by rotating $R_2$ is $6\pi$

Calculate the value of $m$

**[3 marks]**

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**14 (a)** The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 4 & -3 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 5 & 4 \\ -3 & -2 \end{bmatrix}$$

**14 (a) (i)** Find the matrices $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$

**[2 marks]**

**14 (a) (ii)** Hence verify that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

**[2 marks]**

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**14 (b)** Given that $(\mathbf{CD})^{-1} = \mathbf{D}^{-1}\mathbf{C}^{-1}$ is true for all non-singular square matrices **C** and **D**, prove by induction that

$$(\mathbf{M}^{-1})^n = (\mathbf{M}^n)^{-1}$$

is true for all $n \in \mathbb{N}$, where **M** is a non-singular square matrix.

**[4 marks]**

**Turn over ▶**

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**15** The graph $G_1$ has the polar equation

$$r = \frac{8}{\cos\theta + \sqrt{3}\sin\theta} \qquad -\frac{\pi}{6} < \theta < \frac{5\pi}{6}$$

The graph $G_2$ has the polar equation

$$\theta = \frac{2\pi}{3}$$

**15 (a)** Show that $G_1$ is a straight line by writing it as a Cartesian equation in the form $y = mx + c$ where $m$ and $c$ are constants to be found.

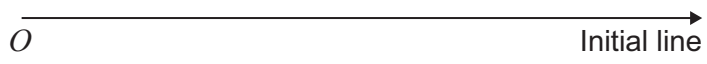
**[3 marks]**

**15 (b)** On the initial line shown in **Figure 1**, sketch $G_1$ and $G_2$

Include the polar coordinates of the intersection point of $G_1$ with the initial line.

**[3 marks]**

**Figure 1**



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**15 (c)** Find the polar coordinates of the intersection point of $G_1$ and $G_2$

**[3 marks]**

**15 (d)** Find the area of the region bounded by the initial line and graphs $G_1$ and $G_2$

**[2 marks]**

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**16** A circle $C$ is drawn on an Argand diagram.

The roots of the equation $w^2 + 4w + 9 = 0$ lie on $C$

**16 (a)** Show that the roots of the equation

$$w^2 + 4w + 9 = 0$$

are

$$-2 + \mathrm{i}\sqrt{5} \quad \text{and} \quad -2 - \mathrm{i}\sqrt{5}$$

**[2 marks]**

**16 (b)** Explain briefly why the centre of $C$ must lie on the real axis.

**[1 mark]**

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**16 (c)** The point $7 + \mathrm{i}\sqrt{14}$ also lies on $C$

**16 (c) (i)** Find the real number which represents the centre of $C$

**[2 marks]**

**16 (c) (ii)** Find the equation of $C$

Give your answer in the form $|z - a| = b$ where $a$ and $b$ are constants.

**[3 marks]**

**END OF QUESTIONS**

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